This question paper contains 8 printed pages]

Your Roll No. ....

5180

B.Sc. (PHYSICAL SCIENCE)/Ist Sem. B

Paper MAPT-101

Mathematics-I (Calculus and Matrics)

(Admission of 2010 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* questions from each Section.

Section I

1\. (*a*) Verify that the set

$$\left\{\begin{bmatrix}\pi \\ 0\end{bmatrix}, \begin{bmatrix}1 \\ e\end{bmatrix}\right\}$$

of vectors is a basis of R^2. 6

(*b*) Examine which of the following is a subspace of R^2. If it is a subspace, give its geometric interpretation :

$$V_1 = \{(a, 2a) : a \in R\}$$

$$V_2 = \{(a, b) : a > 0, a, b \in R\}.$$ 6

2\. (*a*) Let $T : R^3 \to R^2$ be a linear transformation such that $T(1, 0, 1) = (2, -1)$, $T(0, 1, 1) = (1, 1)$, and $T(1, 1, 0) = (-1, 4)$. Find $T(1, 1, 1)$. 6

(*b*) Let R be the rectangle with vertices (1, 1), (1, 4), (3, 1) and (3, 4). Determine and sketch the image of R under :

(*i*) a reflection about the *y*-axis.

(*ii*) a translation by vector (1, 1). 6

3\. (*a*) Reduce the matrix

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 3 & 6 & 12 \\ 2 & 4 & 8 \end{bmatrix}$$

to triangular form by elementary now operations and hence determine its rank. 6

(*b*) Find the characteristic equation, eigen values and eigen vector corresponding to one of them for the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}.$$ 6

4. (*a*) Solve the system of equations :

$$x + 2y - z = 3$$

$$3x - y + 2z = 1$$

$$2x - 2y + 3z = 2.$$ 6

(*b*) For what values of λ and μ do the following system of equations :

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

have an infinite number of solutions. 6

Section II

5. (*a*) Discuss the convergence of a sequence :

$$\left\langle \frac{n}{3n+1} \right\rangle.$$ 6

(*b*) Find the *n*th derivative of

$$y = \frac{x+1}{x^2 - 4}.$$ 6

(*c*) If

$$y = e^{m \sin^{-1} x},$$

show that :

$$\left(1 - x^2\right) y_{n+2} - (2n+1) x y_{n+1} - \left(n^2 + m^2\right) y_n = 0.$$ 6

6. (*a*) Sketch the graph of a function :

$$f(x) = -(x + 2)^2 + 3.$$ 6

(*b*) Find the Maclaurin series expansion of $y = \cos 2x$, assuming that :

$$\lim_{n \to \infty} R_n(x) = 0.$$ 6

(c) In a school, there are 1000 students and all are likely to get infected with eye-flu virus. Initially, 20 students got infected and within 2 weeks, 100 students got infected with the disease. In how much time would the majority of students be infected by the eye-flu virus ? It is given that the disease spreads with logistic growth model. 6

7. (a) Draw the level curves of height $k = 0, 1, 2$ for the surface :

$$z = f(x, y) = 5\sqrt{\frac{x^2}{16} + \frac{y^2}{9} - 1}.$$ 6

(b) If $v = r^m$, where $r^2 = x^2 + y^2 + z^2$, show that :

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = m(m+1)\, r^{m-2}.$$ 6

(c) Verify that the function :

$$w = \cos(5x + 5ct)$$

is a solution of the wave equation. 6

8. (a) For what values of x can we replace $\sin x$ by $x - \frac{x^3}{6}$ with an error of magnitude no greater than 3×10^{-4}. 6

(b) Verify which of the following sequences are monotonic and bounded :

(i) $\left\langle \frac{2^n}{n!} \right\rangle$ 6

(ii) $\left\langle n - 2^n \right\rangle$. 6

Section III

9. (a) Find the radius and centre of the circle whose equation is :

$$z\bar{z} - (2 + 3i)z - (2 - 3i)\bar{z} + 9 = 0.$$ 3½

(*b*) Form an equation in the lowest degree with real coefficients which has $2 - 3i$ and $3 + 2i$ as two of its roots. 4

10. (*a*) Find the modulus and argument of the centroid of the triangle whose vertices are given by $8 + 5i$, $-3 + i$ and $-2 - 3i$, respectively. 3½

(*b*) Let z_1, z_2, z_3 be complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$. Prove that :

$$z_1^2 + z_2^2 + z_3^2 = 0.$$ 4

11. (*a*) If $x = \cos\theta + i\sin\theta$ and $y = \cos\phi + i\sin\phi$, prove that :

$$\frac{x-y}{x+y} = i\tan\frac{\theta-\phi}{2}.$$ 4

(*b*) Use De Moivre's theorem to solve the equation :

$$z^7 + z = 0.$$ 3½

12. (*a*) Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 1 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

using elementary operations. 4

(*b*) Express vector $X = (3, 1, -4)$ as a linear combination of the vectors $X_1 = (1, 1, 1)$, $X_2 = (0, 1, 1)$ and $X_3 = (0, 0, 1)$. 3½